Study of the energy efficiency of the fractional Duffing system with positive linear elasticity

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The Duffing system is often analyzed as a damper of unwanted vibrations or as a system designed for energy harvesting, both in the form of integer-order [1, 2],and fractional [3, 4] systems. The current work presents the results of numerical studies of the energy efficiency of a fractional Duffing system depending on its fractionality, i.e. fractional orders of ordinary derivatives occurring in the differential equations describing the system. The Grünwald-Letnikov method is adopted to solve the system of fractional equations [5].

Numerical solutions of the system under study are used to calculate powers absorbed or dissipated by particular components of the system. Then, the cumulative energies of the system components are calculated. To assess the quality of the solution of the fractional system, a method was developed that refers to the law of energy conservation. Additionally, a solution quality assessment coefficient is proposed, calculated on the basis of the accumulated energy.

The paper presents the results of cumulative energy calculations for systems vibrating in stationary modes and for non-stationary cases in which the value of the excitation frequency is swept in two directions. It has been shown that for some modifications of the fractional system, a significant extension of the frequency range can be achieved, in which the energy efficiency of the system is significantly high compared to the integer-order classical system.

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