Optimal Control: a Journey in Sweeping Systems and Approximations

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In the last decade, some important breakthrough has ben accomplished regarding the derivation of optimality conditions for optimal control problems involving sweeping systems. Sweeping systems can be seens as systems of the form

$$\dot{x}(t) \in f(t, x(t), u(t)) - N_{C(t)}(x(t)),\ u(t) \in U, \quad x(0) \in C_0.$$

where $N_C(t)(x(t))$ is the normal cone to a set $C(t)$ at the point $x(t)$.

They were introduced in the seminal paper [6] by J.J. Moreau in the context of plasticity and friction theory. Recently, there has been an interest in such systems because of their noble applications rainging from problems from mechanics, engineering, economics and crowd motion problems. Optimal control problems problems involving such systems have long been a challenge because of the presence of discontinuous differential inclusions. Latetly, there has been considerable research on optimal control problem involving controlled sweeping systems. In this respect, we refer the reader to, for example, [1], [2], [3], [4] and [5]. This talks centers on approximations techniques such optimal control problems that were first introduced in [7] (see also [8]). We discuss the pros and cons of the approximation scheme used in [7], [9] and [10]. And we also consider different applications of analogous approximation techniques.

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