The role of M-matrices in the study of nonlinear operator systems via monotone operators methods

Michał Bełdziński

In the talk I will study a system of equations of the form

$$\begin{cases} -\operatorname{div}\big(\varphi_1\left(x, \|\nabla u(x)\|\right) \nabla u(x)\big) + \langle \boldsymbol{a} \mid \nabla u(x)\rangle = f_1\big(x, u(x), w(x)\big) & \text{for } x \in \Omega, \\ -\operatorname{div}\big(\varphi_2\left(x, \|\nabla w(x)\|\right) \nabla w(x)\big) + \langle \boldsymbol{b} \mid \nabla w(x)\rangle = f_2\big(x, u(x), w(x)\big) & \text{for } x \in \Omega, \\ u(x) = w(x) = 0 & \text{for } x \in \partial\Omega. \end{cases} \tag{1}$$

and present conditions that guarantee the existence of solutions of the system (1) that are also (in a particular, potential case, when $\boldsymbol{a} = \boldsymbol{b} = \boldsymbol{0}$) Nash-type equilibrium for functionals

$$E_1(u, w) = \int_\Omega \int_0^{\|\nabla u(x)\|} \varphi_1\left(x, s\right) s \; dsdx - \int_\Omega \int_0^{u(x)} f\big(x, s, w(x)\big) dsdx,$$
$$E_2(u, w) = \int_\Omega \int_0^{\|\nabla w(x)\|} \varphi_2\left(x, s\right) s \; dsdx - \int_\Omega \int_0^{w(x)} f\big(x, u(x), s\big) dsdx,$$
$$\text{for all } u, w \in H_0^1(\Omega)$$

An investigation into the existence of such solutions, started in [4], was continued for instance in [3, 5]. All the works mentioned were based on Perov's Contraposition Principle theorem and on Ekelnand's Variational Principle or on other variational techniques. Presented approach, using M-matrices and the Theory of Monotone Operators, developes our earlier results, see [1, 2].

References

[1] M. Bełdziński, M. Galewski, *Nash-type equilibria for systems of non-potential equations*, Appl. Math. Comput. 385, 9 (2020)

[2] M. Bełdziński, M. Galewski, D. Barilla, *Nash-type equilibria for systems of partially potential nonlinear equations*, Math. Methods Appl. Sci. 46, No. 11, 11830–11841 (2023)

[3] A. Budescu, R. Precup, *Variational properties of the solutions of singular second-order differential equations and systems*, J. Fixed Point Theory Appl. 18, No. 3, 505–518 (2016)

[4] R. Precup, *Nash-type equilibria and periodic solutions to nonvariational systems*, Adv. Nonlinear Anal. 3, No. 4, 197–207 (2014)

[5] R. Precup, A. Stan, *Linking methods for componentwise variational systems*, Result. Math. 78, 25 (2023)

First Author: Michał Bełdziński
Affiliation: *Institute of Mathematics, Lodz University of Technology 93-590, Poland*
e-mail: `michal.beldzinski@p.lodz.pl`