Stochastic Hall-magneto-hydrodynamics equations

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Magnetohydrodynamics describes the motion of electrically conductive fluid in the presence of a magnetic field. We consider the following stochastic Hall-MHD system

$$
\begin{aligned}
d\mathbf{u} + \Big[(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p - s\,(\mathbf{B}\cdot\nabla)\mathbf{B} + s\,\nabla\Big(\frac{|\mathbf{B}|^2}{2}\Big) - \nu_1\,\Delta\mathbf{u}\Big]\,dt \;&=\; \mathbf{G}_1(t,\mathbf{u})dW_1(t),\\
d\mathbf{B} + \Big[(\mathbf{u}\cdot\nabla)\mathbf{B} - (\mathbf{B}\cdot\nabla)\mathbf{u} + \varepsilon\,\mathrm{curl}[(\mathrm{curl}\mathbf{B})\times\mathbf{B}] - \nu_2\,\Delta\mathbf{B}\Big]\,dt \;&=\; \mathbf{G}_2(t,\mathbf{B})dW_2(t)
\end{aligned}
$$

with the incompressibility conditions $\mathrm{div}\,\mathbf{u}=0$ and $\mathrm{div}\mathbf{B}=0$ and appropriate initial conditions. In this problem $\mathbf{u}$, $\mathbf{B}$ and p represent velocity, magnetic field and pressure, respectively. The terms $\mathbf{G}_1(t,\mathbf{u})dW_1(t)$, $\mathbf{G}_2(t,\mathbf{B})dW_2(t)$, where $W_1(t), W_2(t)$ are cylindrical Wiener processes, stand for random forces. We concentrate on the existence of a global martingale solution. The construction of the solution is based on the Fourier analysis, the stochastic compactness method and Jakubowski's generalization of the Skorokhod theorem for non-metric spaces.

References

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