Applications of the Banach fixed point theorem to analyze insolvency problems of an insurance company

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Abstract

We explore new areas of applications of the Banach fixed point theorem to solve some problems arising in mathematical economics with a special attention paid to the insurance mathematics and the solvency challenges faced by insurance companies in their daily business activities. We use Banach's contraction principle to construct sequences of new monotone upper and lower bounds for the deficit distribution at ruin $\Psi(u,\, y)$ of the insurer, considered as a function of the initial surplus (capital) u and the severity of ruin y. From the mathematical perspective, a direct application of this principle does not seem to be immediate, since existing risk operators have usually infinitely many fixed points. A question arises: how to make them contractions? We have solved this problem by considering a monotone risk operator L on a properly defined complete metric space $\langle \mathcal{R}, d_r \rangle$ in which $\Psi(u,\, y)$ is shown to be the unique fixed point. What is more, L is proven to be a contraction on $\langle \mathcal{R}, d_r \rangle$. The resulting procedure enables the insurer to approximate $\Psi(u,\, y)$ (and effectively control the error of approximation) by iterating L on any point from $\langle \mathcal{R}, d_r \rangle$. The approach presented in this paper enables to treat in a unified way some discrete and continuous time insolvency risk models.

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