## Dynamical Analysis of Optical Soliton Patterns in the Flexibly Supported Euler-Bernoulli Beam Equation: A Semi-Analytical Solution Approach

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The study investigate the non-linear vibration of a flexibly supported Euler-Bernoulli beam, a 1+1-dimensional system subjected to axial loads. The modified Khater method, a pivotal tool in mechanical engineering, is employed for analysis of analytical solutions of the wave form being part of each solution. Notably, periodic soliton solutions for the nonlinear model closely align with numerical and approximate analytical solutions, showcasing the accuracy of our modeling approach. Density diagrams, contour diagrams, Poincaré sections in two or three dimensional space depicting the obtained analytical solutions are presented, elucidating the accuracy provide visual confirmation of the engineering model's physical significance. Utilizing mathematical models and appropriate parameter values, the planar dynamical system is derived through the Galilean transformation, further optimizing our understanding. We elucidate sensitivity analysis and phase portraits with equilibrium points are illustrated by analyzing a special case of the investigated dynamical system highlighting its symmetrical properties. Finally, we use sensitivity analysis and for the observation of sensitivity and also identify quasi-periodic, chaotic and periodic behaviors with an extra weak forcing term confirmed by Poincaré sections and Lyapunov exponents spectra.

## References

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