

Boundary value problems with non-local conditions

Piotr Kasprzak

In the last decades, nonlocal boundary value problems have become a rapidly growing area of research. The interest is driven not only by theoretical curiosity but also by the practical applicability of such problems in various fields including engineering, physics, and life sciences. In a very general formulation, a second-order nonlinear equation with nonlocal boundary conditions has the form

$$\begin{aligned}x''(t) + p(t)x'(t) + q(t)x(t) + r(t)g(t, x(t)) &= 0, \quad t \in [0, 1], \\ax(0) - bx'(0) &= \alpha[x], \quad cx(1) + dx'(1) = \beta[x],\end{aligned}$$

where $p, q, r: [0, 1] \rightarrow \mathbb{R}$ and $g: [0, 1] \times \mathbb{R} \rightarrow \mathbb{R}$ are given functions and $\alpha, \beta: C[0, 1] \rightarrow \mathbb{R}$ are linear functionals which are expressed by the Riemann–Stieltjes integrals.

During the talk, I will present some recent existence results for BVPs with nonlocal boundary conditions. Special emphasis will be placed on highlighting the role of functions of bounded variation in the theory. Additionally, I will provide examples to illustrate the abstract results.

References

- [1] J. Appell, D. Bugajewska, S. Reinwand, *Nonlocal boundary value problems with BV-type data*, Electron. J. Qual. Theory Differ. Equ. **2020** (2020), paper no. 69, 18 pages, DOI: 10.14232/ejqtde.2020.1.69.
- [2] D. Bugajewska, G. Infante, P. Kasprzak, *Solvability of Hammerstein integral equations with applications to boundary value problems*, Z. Anal. Anwend. **36** (2017), no. 4, 393–417, DOI: 10.4171/ZAA/1594.
- [3] D. Bugajewska, J. Mawhin, *Boundary value problems with bounded φ -Laplacian and nonlocal conditions of integral type*, Czech Math J (2023), 12 pages, DOI: 10.21136/CMJ.2023.0154-23.

First Author: Piotr, Kasprzak

Affiliation: Faculty of Mathematics and Computer Science,
Adam Mickiewicz University in Poznań
61-255 Poznań, Poland

e-mail: kasp@amu.edu.pl