Population of entities with three individual states and asymmetric interactions

Mirosław Lachowicz, Milena Matusik, Krzysztof A. Topolski

In various applications in life and social sciences the agents of a complex system, which can be individuals of a sub–populations, can have one of the few inner states or can choose one of few strategies. Their states are effects of interactions with other agents. Behavior is described by a kinetic–like a nonlinear equation. We study the behavior of solutions in the case of three possible states and show that in some cases a kind of self–organization occurs but in others, a periodic behavior characterizes the system. We can observe a wide variety of dynamics that can relate to the behavior of real systems. The model contains a natural interaction intensity parameter $\gamma \geq 1$. Case $\gamma > 1$ leads to "*very*" nonlinear structures. We prove that the system has no non–constant periodic solutions. We propose conditions guaranteeing the asymptotic stability of equilibrium points on the boundary that corresponds to the asymptotic extinction of two states. Moreover, conditions for the uniqueness and instability of an inner equilibrium point, corresponding to an asymptotic presence of all states, are formulated. The case of $\gamma = 1$ with asymmetric interaction rate is studied as well. Possible complex behavior, also periodic, of solutions can reflect the possible complex performance of systems with asymmetric interactions — typical e.g. in Economy and some applications in Biology.

References

[1] M. Lachowicz, M. Matusik, K.A. Topolski, *Population of entities with three individual states and asymmetric interactions*, Appl. Math. Comp. 464 (2024) 128395

[2] M. Parisot, M. Lachowicz, *Kinetic model for the formation of swarms with nonlinear interactions*, KRM 9 (1), 131–164 (2016)

First Author: Mirosław Lachowicz
Affiliation: *Institute of Applied Mathematics and Mechanics, Faculty of Mathematics, Informatics and Mechanics, University of Warsaw 02-097 Warsaw, Poland*
e-mail: lachowic@mimuw.edu.pl

Second Author: Milena Matusik
Affiliation: *Institute of Mathematics, University of Gdańsk 80-308 Gdańsk, Poland*
e-mail: milena.matusik@ug.edu.pl

Third Author: Krzysztof A.Topolski
Affiliation: *Department of Mathematics and Physics, Polish Naval Academy 81-127 Gdynia, Poland*
e-mail: k.topolski@amw.gdynia.pl