Necessary optimality condition for Lagrange problem with fractional partial system

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In paper [1] the authors introduce the concept of fractional partial derivatives $D_x^{\alpha}z$, $D_y^{\beta}z$, $D_{x,y}^{\alpha,\beta}z$ and consider the following continuous Fornasini-Marchesini system of fractional order

$$D_{x,y}^{\alpha,\beta}z(x,y) = f(x,y,z(x,y),D_x^{\alpha}z(x,y),D_y^{\beta}z(x,y),u(x,y)), \tag{1}$$

for $(x,y) \in P = [0,a] \times [0,b]$ a.e., with initial conditions

$$\begin{cases} I_{x,y}^{1-\alpha,1-\beta}z(x,0) = \gamma(x), \ x \in [0,a] \\ I_{x,y}^{1-\alpha,1-\beta}z(0,y) = \delta(y), \ y \in [0,b] \end{cases} \tag{2}$$

where α, $\beta \in (0,1)$. The main results are existence, uniqueness and continuous dependence of solutions to (1)–(2) on functional parameter (control) $u : P \to \mathbb{R}^m$.

The research concerning the above system is continuing in paper [2], where the Lagrange problem with a nonlinear cost functional of the form

$$\mathcal{J}(z,u) = \iint_P F\left(x,y,z(x,y),D_x^{\alpha}z(x,y),D_y^{\beta}z(x,y),u(x,y)\right)dxdy$$

and system (1)–(2) is invesitagated. The main result of [2] is a sufficient condition for the existence of optimal solution to the above Lagrange problem based on the convexity of a certain set.

In the presented paper we formulate a necessary condition for the existence of optimal solution to the same Lagrange problem applying Pontryagin's maximum principle.

References

[1] D. Idczak, R. Kamocki, and M. Majewski. *Nonlinear continuous Fornasini–Marchesini model of fractional order with nonzero initial conditions.* Journal of Integral Equations and Applications, 32(1), 19–34 (2020).

[2] M. Majewski. *On the existence of optimal solutions to the Lagrange problem governed by a nonlinear Goursat-Darboux problem of fractional order.* Opuscula Mathematica, 43(4), 547–558 (2023).

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