

Multivariate Hawkes Processes and Markovianizations

Mariusz Niewegłowski

A very interesting and important class of stochastic processes was introduced by Alan Hawkes in [1]. These processes, called now Hawkes processes, are meant to model self-exciting and mutually-exciting random phenomena that evolve in time. The self-exciting phenomena are modeled as univariate Hawkes processes, and the mutually-exciting phenomena are modeled as multivariate Hawkes processes. The Hawkes processes have been applied to modeling in many areas of science, including: insurance, finance, seismology and neurology. In this talk we provide some results on markovianity of the Generalized Multivariate Hawkes Processes (GMHP) introduced in our earlier papers. GMHP are multivariate marked point processes that add an important feature to the family of the (classical) multivariate Hawkes processes: they allow for explicit modelling of simultaneous occurrence of excitation events coming from different sources, i.e. caused by different coordinates of the multivariate process. We propose that this structure of mutual excitations is specified in terms of the excitation graph. We provide results which show that under some conditions on its kernels the intensities of GMHP's are functions of a Markov processes. Moreover we show that it is possible to compute their Laplace transform and some moments by means of systems of ODE's.

References

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First Author: Mariusz, Niewegłowski

Affiliation: *Faculty of Mathematics and Information Science,
Warsaw University of Technology
00-661, Poland*

e-mail: mariusz.niewegłowski@pw.edu.pl