On the use of Lie brackets in the presence of state constraints

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The classical *inward pointing condition* (IPC) for a control system whose state x is constrained in the closure $C := \bar{\Omega}$ of an open set Ω prescribes that at each point of the boundary $x \in \partial\Omega$ the intersection between the dynamics and the interior of the tangent half-space of $\bar{\Omega}$ at x is nonempty. Under this hypothesis, for every system trajectory $x(.)$ on a time-interval $[0,T]$, possibly violating the constraint, one can construct [3] a new system trajectory $\hat{x}(.)$ that satisfies the constraint and whose distance from $x(.)$ is bounded by a quantity proportional to the maximal deviation $d := \text{dist}(\Omega, x([0,T]))$. When (IPC) is violated, the construction of such a constrained trajectory is not possible in general. However, in [1] we prove that a "higher order" inward pointing condition involving Lie brackets of the dynamics' vector fields –together with a non-positiveness curvature-like assumption– allows for a novel construction of a constrained trajectory $\hat{x}(.)$ whose distance from the reference trajectory $x(.)$ is bounded by a quantity proportional to $\sqrt{d}$. As an application, we establish the continuity *up to the boundary* of the value function V of a connected optimal control problem, a continuity that allows to regard V as the *unique* constrained viscosity solution of the corresponding Bellman equation.

References

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