Existence of nonoscilatory solution on k-dimensional system of delayed nonlinear discrete equations with p-Laplacian

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We consider the k-dimensional system of delayed discrete nonlinear equations with p-Laplacian in the following form

$$\begin{cases} \Delta\phi_p\left(x_i(n) + q_i(n)\, x_i(n-l_i) - C_i\right) = a_i(n)\, f_i(x_{i+1}(n-m_i)) + b_i(n), \\ \Delta\phi_p\left(x_k(n) + q_k(n)\, x_k(n-l_k) - C_k\right) = a_k(n)\, f_k(x_1(n-m_k)) + b_k(n), \end{cases}$$

where $i = 1, \dots, k-1$, $n \in \mathbb{N}_0 = \{n_0, n_0+1, \dots\}$, $n_0 = \max_{i=1,\dots,k}\{l_i, m_i\}$, $l_i, m_i \in \mathbb{N} = \{0, 1, 2, \dots\}$. Here Δ is the forward difference operator defined by $\Delta u(n) = u(n+1) - u(n)$, and ϕ_p is p-Laplacian defined by $\phi_p(t) = |t|^{p-1}t$, $p > 1$, $t \in \mathbb{R}$. Moreover $q_i = (q_i(n))$, $a_i = (a_i(n))$, $b_i = (b_i(n))$ are given sequences of real numbers, C_i are given constants, $f_i\colon \mathbb{R} \to \mathbb{R}$ are given functions, and $x_i = (x_i(n))$ for $i = 1, \dots, k$ are unknown real sequences. Throughout this paper X denotes an unknown vector $(x_1, \dots, x_k)$ and $X(n)$ denotes $(x_1(n), \dots, x_k(n)) \in \mathbb{R}^k$. The aim of this paper is to present sufficient conditions for the existence of bounded positive persistent solutions of the above system with various $(q_i(n))$, $i = 1, \dots, k$. The main tool used in the proofs of presented theorems is Krasnoselskii's Fixed Point Theorem.

References

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