Applications of dynamical systems to water waves

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The topic of water waves is an old one and one can say that the theory of water waves was initiated almost two hundred years ago. The classical problem of water waves is the problem consisting in solving the incompressible Euler equations in a domain bounded above by a free surface (the interface between air and water) and below by a solid boundary (the bottom). What makes the water-wave problem difficult is not its governing equation which is linear (Laplace's equation), but its two nonlinear boundary conditions on the free surface.

In this talk, we show how dynamical systems methods can be used to obtain results on the spatial behavior of travelling waves near the undisturbed free surface state. The idea to use "dynamical" arguments for solving nonlinear elliptic problems in a strip was developed in the 1980s, pioneered by Kirchgässner. Later Iooss and his collaborators developed further the ideas, without taking advantage of the Hamiltonian structure of the spatial water-wave problem (see the work of Bridges for example) but only using the framework of reversible vector fields. We will also show how some mathematical results have led to the discovery of new solutions, which then helped us to get a better insight into the physics of water waves.

References

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