Thin layer approximation for a coupled bulk-surface PDE

Adam Bobrowski, Anotida Madzvamuse and Elżbieta Ratajczyk

Motivated by recent biological findings on Rho GTPases's self-organisation and pattern formation, we study a system of coupled bulk-surface partial differential equations, describing changes in concentration of these proteins in a cell. We show that the corresponding Cauchy problem is well-posed in an L^1-type space and is governed by a Markov semigroup generator. Additionaly, the corresponding Kolmogorov backward equation describes dynamics of a Feller process.

Next, since the cell cortex is typically rather thin, we investigate the limit as the thickness converges to 0 and establish the convergence theorems for both Markov and Feller semigroups involved. To this end, we observe that letting the thickness of the annulus to zero is equivalent to keeping it constant but increasing the rate of radial diffusion. Consequently, an appropriate rescaling of coefficients and finding a common reference space allows deriving the form of the limit equation. The bulk solutions gradually lose dependence on the radial variable and in the limit may be regarded as functions on the circle. Thus the limit equations can be seen as describing surface diffusion on two copies of the circle with jumps from one copy to the other: transmission conditions featuring in the approximating equations become integral parts of the limit master equation.

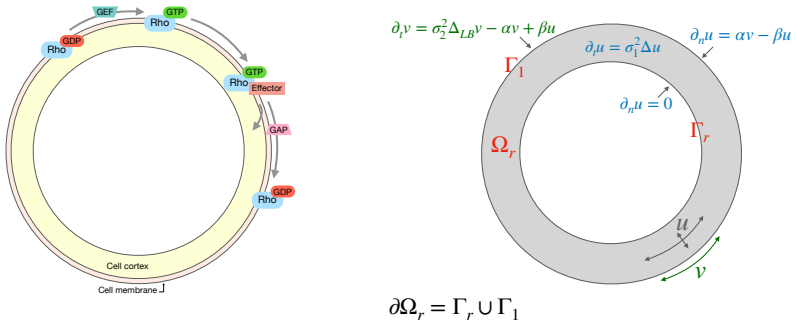

Figure 1: A schematic representation of the Rho GTPase cyclic spatiotemporal interactions between *active* (v) and *inactive* (u) Rho GTPases.

References

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First Author: Adam Bobrowski
Affiliation: *Department of Mathematics, Lublin University of Technology 20-618 Lublin, Poland*
e-mail: a.bobrowski@pollub.pl

Second Author: Anotida Madzvamuse
Affiliation: *Mathematics Department, The University of British Columbia, Vancouver, BC Canada V6T 1Z2*
e-mail: am823@math.ubc.ca

Third Author: Elżbieta Ratajczyk
Affiliation: *Department of Mathematics, Lublin University of Technology 20-618 Lublin, Poland*
e-mail: e.ratajczyk@pollub.pl