Time Optimal Control of Ermakov's Equation

Heinz Schättler and Dionisis Stefanatos

The global solution to the following time-optimal control problem is derived [4]: given $a, b \geq 1$, minimize the transfer time T from $z(0) = 1$ into $z(T) = \gamma > 1$ over all Lebesgue measurable functions $u : [0, T] \to [-a, b]$ subject to the dynamics $\ddot{z} + uz - \frac{1}{z^3} \equiv 0$. This second-order ODE, called Ermakov's equation [1], models frictionless atom cooling in a harmonic trap [2, 3]. Any of its solutions for the boundary value problem for z achieves a temperature reduction by the factor $\frac{1}{\gamma}$.

References

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First Author:	Heinz Schättler
Affiliation:	*The Preston M. Green Dept. of Electrical and Systems Engineering*
	Washington University
	St. Louis, Mo, 63130 USA
e-mail:	hms@wustl.edu

Second Author:	Dionisis Stefanatos
Affiliation:	*Materials Science Department, School of Natural Sciences*
	University of Patras
	26504 Patras, Greece
e-mail:	dionisis@post.harvard.edu