Nonexistence results for fractional differential inequalities

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We will prove nonexistence of global solution of a Caputo fractional differential problem of the form $D_*^\alpha u(t) = \lambda t^\beta |u(t)|^p,\ u(0) = u_0 > 0$ for $0 < \alpha < 1$ and $\alpha + \beta > 0$ when $p > 1$ (β can be negative). This is motivated by work of Laskri and Tatar, Comput. Math. Appl. (2010) and Shan and Lv, Filomat (2024). Laskri and Tatar show that $p_0 := \frac{1+\beta}{1-\alpha}$ is a critical exponent in the sense that for $1 < p < p_0$ there do not exist global solutions, whereas for $p > p_0$ they give an example of a non-zero global solution. We prove that global solutions do not exist for all $p > 1$. The reason for these differences is that we have $u_0 > 0$, their example has $u_0 = 0$. We also improve on the result in Shan and Lv paper by considering a more general problem and by giving a more precise conclusion.

References

[1] Y. Laskri and N-e Tatar, *The critical exponent for an ordinary fractional differential problem.* Comput. Math. Appl. 59 (2010), no. 3, 1266-1270.

[2] Y. Shan and G. Lv, *New criteria for blow-up of fractional differential equations.* Filomat 38:4 (2024), 1305-1315.

[3] J. R.L. Webb, *Nonexistence results for fractional differential inequalities.* Submitted for publication.

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