Dynamical System for Tolman-Oppenheimer-Volkoff Equation

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We extend in [2] our results published in Comm. Math. Phys. 2021 [9] and J. Diff. Eqs 2023 [5] to cover relativistic case similarly as in Math. Meth. Appl. Sci. 2023 [4] modelling dark matter model for Tolman-Oppenheimer-Volkoff Equation

$$-rc^2p'(r)(rc^2-2Gm(r))=G(c^2m(r)+4\pi r^3p(r))(c^2\rho(r)+p(r))$$

as an alternative to black hole model studied recently by Klainerman, Szeftel and Giorgi [6] and Dafermos, Holzegel, Rodnianski and Taylor [7] both in static Schwarzschild and rotating Kerr geometries. For the introduction see the review papers of Giorgi and Bieri. The results obtained by Genzel and Ghez for Sagittarius A* were analyzed by Ruffini [8] and Chavanis [3] in the framework of dark matter with the modified relativistic Michie-King distribution function yielding the relevant equation of state providing the energy momentum tensor for Einstein equation. We analyze the dynamical system for which the global Lyapunov function is obtained thus yielding the limit mass for the system with gravitational collapse into a black hole.

References

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