Analysis of a Mathematical Model for Low-Grade Glioma under Chemotherapy as a Dynamical System

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We analyze dynamical system properties of a clinically validated mathematical model [1] describing the growth of Low-grade Glioma under chemotherapy. It is assumed that a cell-cycle non-specific drug damages both proliferating and quiescent cells, but possibly at different rates. While damage to proliferating cells is lethal, damaged quiescent cells may be repaired and can reenter the cell cycle. We prove that there exists a unique dosage $\tilde{u}$ (depending only on the parameters of the dynamics of the system) such that the tumor can be eradicated for $u \geq \tilde{u}$ as all trajectories converge to the tumor-free equilibrium point (global stability). For lower doses, $0 \leq u < \tilde{u}$, however, there exists a locally asymptotically stable equilibrium point with positive values and such doses are not sufficient to eradicate the tumor. Mathematically, for $u = \tilde{u}$, the positive and tumor-free equilibrium points are equal and a trans-critical or exchange of stability bifurcation occurs. Our theoretical analysis is independent of specific values of the parameters [2]. For the numerical illustration of the results we use clinically validated parameter values for Low-grade Glioma from [1]. This analysis provides the first step towards optimizing treatment protocols for this disease.

References

[1] B. Ribba, G. Kaloshi, M. Peyre et al., *A tumor growth inhibition model for Low-Grade Glioma treated with chemotherapy or radiotherapy*, Cancer Therapy: Clinical, **18**(18), 5071–5080, 2012, doi: 10.1158/1078-0432.CCR-12-0084

[2] U. Ledzewicz and H. Schättler, *Analysis of a mathematical model for Low-grade Glioma under chemotherapy as a dynamical system*, Nonlinear Analysis RWA (Real World Applications), Special issue dedicated to the memory of V. Lakshmikantham, to appear

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