Minimal solutions for a certain class of elliptic problems in exterior domains

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The presentation considers the problem of finding sufficient conditions to prove that the equation

$$\begin{cases} \triangle u(x) + f(x, u(x)) + g(x, u) \cdot x \cdot \bigtriangledown u(x) = 0 \quad \text{for} \quad x \in \Omega_R \\ \lim\limits_{\|x\| \to +\infty} u(x) = 0 \end{cases} \tag{1}$$

has a classical positive solution in exterior domain $\Omega_R = \{x \in \mathbb{R}^n, \|x\| > R\}$ for some real number $R > 1$ and integer $n > 2$. Our main method is the subsolution-supersolution method based on Noussair-Swanson theorem, which is applied to show the existence of a solution of (1). We will construct the supersolution as a radial solution of some elliptic auxiliary problems. In this part we will use the fixed point theorems. We also want to describe how the solution behaves when norm of x tends to infinity. All above considerations are checked with f positive at the origin ('positone problem'). In the second part we consider the case when $g(x, u) = g(x) \cdot u^{q_2}$ and our nonlinearity is negative at the origin ('semipositone problem').

References

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