The dynamics and integrability of multiple pendula

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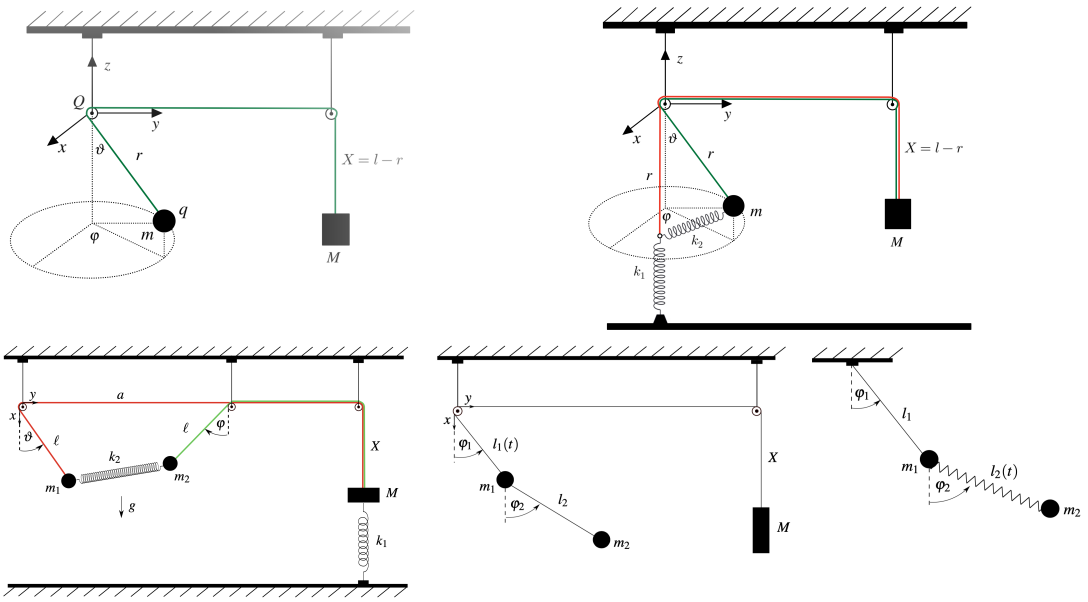
We investigate the dynamics and integrability of various types of pendulum systems. As Hamiltonian systems with three degrees of freedom, their analysis presents a significant challenge. To gain insight into the systems' dynamics, we employ various numerical methods, including Lyapunov exponent spectra, phase-parametric diagrams, and Poincaré cross-sections. The novelty of our work lies in the integration of these three numerical methods into one powerful tool. We provide a comprehensive understanding of the systems' dynamics by identifying parameter values or initial conditions that lead to hyperchaotic, chaotic, quasi-periodic, and periodic motion. To the best of my knowledge, this is the first attempt to use the method of Lyapunov exponents in the systematic search for the first integrals of the system. We show how to effectively apply the Lyapunov exponents as an indicator of integrable dynamics. The explicit forms of integrable and superintegrable systems are given.

The numerical analysis is complemented by analytical proofs regarding the systems non-integrability. These proofs are based on the analysis of properties of the differential Galois group of variational equations along specific solutions.



References

[1] W. Szumiński, A. Maciejewski, *Dynamics and non-integrability of the double spring pendulum*, submitted to Journal of Sound and Vibrations, under review.

[2] W. Szumiński, *A new model of variable-length coupled pendulums: from hyperchaos to superintegrability*, Nonlinear Dynamics, 112(6), 4117–4145, 2024.

[3] W. Szumiński, A. Maciejewski *Dynamics and integrability of the swinging Atwood machine generalisations*, Nonlinear Dynamics, 110(21), 2101–2128, 2022.

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