Existence of solutions to nonlinear $2n$th-order discrete boundary value problem with parameter dependence

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We consider the following discrete boundary value problem

$$\Delta^n(p(k)\Delta^n x(k-n)) = \lambda f(k, x(k)), \qquad k \in \mathbb{N}[n, T] \tag{1}$$

$$x(0) = x(1) = \ldots = x(n-1) = 0, \tag{2}$$

$$x(T+1) = x(T+2) = \ldots = x(T+n) = 0, \tag{3}$$

where $\mathbb{N}[n, T] = \{n, n+1, \ldots, T-1, T\}$, λ is a positive parameter and $p : \mathbb{N}[n, T] \to \mathbb{R}$ and $f : \mathbb{N}[n, T] \times \mathbb{R} \to \mathbb{R}$ are known continuous functions. Using variational methods (for instance Mountain Pass Lemma) we present the sufficient conditions for existence the solution of the above problem.

References

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